ELEC350 Assignment 4

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Question 1: Consider a message signal $m(t)$ containing frequency components at 300 and 3000 Hz. This signal is used to make an AM signal with carrier frequency 1 MHz. The receiver has a frequency error of 100 Hz. Determine the frequency components (spectrum) of the AM receiver output. State your assumptions. Repeat for DSB and SSB.

Solution:

Let $f1$ = 300Hz, $f2$ = 3000Hz, ¢f = 100Hz, fc = 1MHz

The message signal is: $m(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$

The AM signal is: $S_{AM}(t) = Ac(1 + kam(t)) \cos(2\pi fct)$

The receiver multiplies $SAM(t)$ with a local oscillator with frequency error Δf. Thus the signal becomes

$$\begin{aligned}
r(t) &= sAM(t) \cos(2\pi(fc + \Delta f)t) \\
&= Ac(1 + m(t)) \cos(2\pi fct) \cos(2\pi(fc + \Delta f)t) \\
&= \frac{A_c}{2}(1 + m(t))(\cos(2\pi(2fc + \Delta f)t) + \cos(2\pi\Delta ft))
\end{aligned}$$

Assuming that the low pass filter would block all of the higher frequency signals, the $r(t)$ becomes

$$\begin{aligned}
r(t) &= \frac{A_c}{2}(1 + m(t)) \cos(2\pi\Delta ft) \\
&= \frac{A_c}{2}(1 + ka(A1 \cos(2\pi f_1 t) + A2 \cos(2\pi f_2 t))) \cos(2\pi\Delta ft) \\
&= \frac{A_c}{2}\cos(2\pi\Delta \mathrm{ft}) + \frac{k_a A_c A_1}{4}\cos(2\pi f_1 t) \cos(2\pi\Delta ft) \\
&\quad + \frac{k_a A_c A_2}{4}\cos(2\pi f_2 t) \cos(2\pi\Delta ft) \\
&= \frac{A_c}{2}\cos(2\pi\Delta ft) + \frac{k_a A_c A_1}{4}[\cos(2\pi(f_1 + \Delta f)t) + \cos(2\pi(f_1 - \Delta f)t)] \\
&\quad + \frac{k_a A_c A_2}{4}[\cos(2\pi(f_2 + \Delta f)t) + \cos(2\pi(f_2 - \Delta f)t)]
\end{aligned}$$

Therefore, the frequencies that would be output from the receiver would be

$$\begin{aligned}
f &= \Delta f \quad \& \quad f_1 \pm \Delta f \quad \& f_2 \pm \Delta f \\
&= 100Hz, 200Hz, 400Hz, 2.9KHz, 3.1KHz
\end{aligned}$$

For a DSB signal ,

$$S_{DSB}(t) = A_c k_a m(t) \cos(2\pi f_c t)$$

The output frequecies would be

$$f = f_1 \pm \Delta f \quad \& \quad f_2 \pm \Delta f$$
$$=200Hz,400Hz,2.9KHz,3.1KHz$$

For a SSB signal,

$$S_{SSB}(t) = \frac{k_a A_c}{4}(A_1 \cos(2\pi(f_1 + f_c t)) + A_2 \cos(2\pi(f_2 + f_c t)))$$

The output frequecies would be

$$f = f_{1+} \Delta f \quad \& \quad f_{2\,+} \Delta f$$
$$=400Hz,3.1KHz$$
or
$$f = f_1 \text{-} \Delta f \quad \& \quad f_2 \text{-} \Delta f$$
$$=200Hz,2.9KHz$$

Question 2a: Draw a block diagram of the general I-Q receiver with input $s(t)$ and outputs $x(t)$,$y(t)$. Include both RF and IF stages. Assume $s(t)$ is an AM signal, and assume a phase difference φ between the transmitted AM carrier and the receiver local oscillator (LO). Show on the diagram how to obtain $m(t)$ from $x(t)$, $y(t)$. Show algebraically that the receiver output is $m(t)$ regardless of the value of φ.
Optional: verify using Matlab.

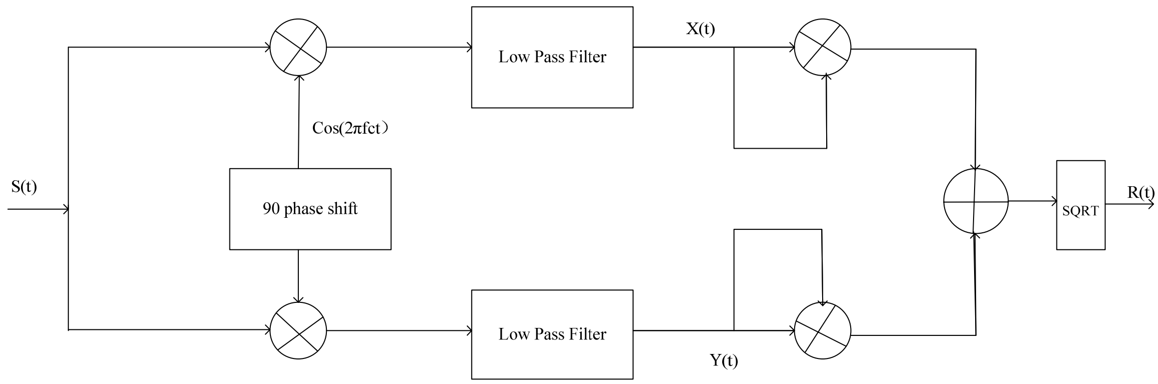

Figure 1: General I-Q Receiver

The AM signal is:

$s_{AM}(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$

The received signal is:

$s(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t + \varphi)$

$= A_c(1 + kam(t))(\cos(2\pi f_c t) \cos\varphi - \sin(2\pi fct) \sin\varphi)$

And

$x(t) = s(t) \cos(2\pi fct)$

$= A_c(1 + k_a m(t))(\cos(2\pi fct) \cos \varphi - \sin(2\pi fct) \sin \varphi) \cos(2\pi fct)$

$= \frac{A_c}{2}(1 + k_a m(t))[(\cos(4\pi fct) + 1) \cos \varphi - \sin(4\pi fct) \sin \varphi]$

After the signal passes through the low pass filter, the remaining component is

$x(t) = \frac{A_c}{2} (1 + k_a m(t)) \cos\varphi$

Similarly,

$y(t) = -\frac{A_c}{2} (1 + k_a m(t)) \sin \varphi$

Thus the output becomes,

$$r(t) = \sqrt{x^2(t) + y^2(t)}$$

$$= \sqrt{\left(\frac{A_c(1 + K_a m(t))}{2}\right)^2 \left(\cos^2(\varphi) + \sin^2(\varphi)\right)}$$

$$= \frac{A_c(1 + K_a m(t))}{2}$$

So, It shows that the receiver output is $m(t)$ regardless of the value of φ.

Question 2b: Draw a schematic diagram with analog components (diodes, resistors, capacitors etc) that can be used to replace the I-Q receiver and still obtain $m(t)$..
Solution:

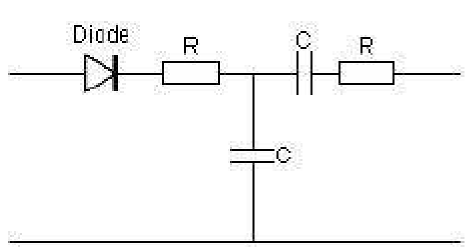

Figure 2: Either half of an I-Q receiver

Question 3: Repeat question 2a assuming a frequency error Δf (but the phase error $\varphi = 0$), and show that the receiver output is still $m(t)$.

The AM signal is :

$$s_{AM}(t) = Ac(1 + k_a m(t)) \cos(2\pi f_c t)$$

$$x(t) = s_{AM}(t) \cos(2\pi(fc + \Delta f)t)$$
$$= Ac(1 + kam(t)) \cos(2\pi fct) \cos(2\pi(fc + \Delta f)t)$$
$$= \frac{A_c}{2}(1 + kam(t))[\cos(2\pi(2fct + \Delta f)t) + \cos(2\pi\Delta ft)]$$

Passing the LPF, it becomes:

$$x(t) = \frac{A_c}{2} (1 + kam(t)) \cos(2\pi\Delta ft)$$

Similarly,

$$y(t) = \frac{A_c}{2} (1 + kam(t)) \sin(2\pi\Delta ft)$$

So,

$$r(t) = \sqrt{x^2(t) + y^2(t)}$$
$$= \frac{A_c(1 + K_a m(t))}{2}$$

So, by removing the DC component, the output is just a constant times $m(t)$

Question 4: Repeat question 3 assuming $s(t)$ is an SSB signal. What is the receiver output ?

$$S_{SSB}(t) = \frac{A_c}{2} (m(t) \cos(2\pi fct) + \hat{m}(t) \sin(2\pi fct))$$

Then,

$$x(t) = S_{SSB}(t) \cos(2\pi(fc + \Delta f)t)$$
$$= \frac{A_c}{2} (m(t) \cos(2\pi fct) + \hat{m}(t) \sin(2\pi fct)) \cos(2\pi(fc + \Delta f)t)$$
$$= \frac{A_c}{4} [m(t)(\cos(2\pi(2fct + \Delta f)t) + \cos(2\pi\Delta ft))$$
$$+ \hat{m}(t) (\sin(2\pi(2fct + \Delta f)t) - \sin(2\pi\Delta ft))]$$

Passing the LPF

$$x(t) = \frac{A_c}{4}\ (m(t)\cos(2\pi\Delta f t) - \hat{m}(t)\sin(2\pi\Delta f t))$$

It shows that the message signal cannot be separated if there is a frequency error Δf.